

# Large- $N_c$ Regge models and the $\langle A^2 \rangle$ condensate

Wojciech Broniowski<sup>1,2</sup> and Enrique Ruiz Arriola<sup>3</sup>

<sup>1</sup>Institute of Nuclear Physics PAN, PL-31342 Cracow, Poland

<sup>2</sup>Institute of Physics, Świętokrzyska Academy, PL-25406 Kielce, Kielce, Poland

<sup>3</sup>Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada  
E-18071 Granada, Spain

Talk presented by WB at the Mini-Workshop Bled 2007:  
HADRON STRUCTURE AND LATTICE QCD, Bled (Slovenia), 9-16 July 2007

## Abstract

We explore the role of the  $\langle A^2 \rangle$  gluon condensate in matching Regge models to the operator product expansion of meson correlators.

This talk is based on Ref. [1], where the details may be found. The idea of implementing the principle of parton-hadron duality in Regge models has been discussed in Refs. [2–8]. Here we carry out this analysis with the dimension-2 gluon condensate present. The dimension-two gluon condensate,  $\langle A^2 \rangle$ , was originally proposed by Celenza and Shakin [9] more than twenty years ago. Chetyrkin, Narison and Zakharov [10] pointed out its sound phenomenological as well as theoretical [11–15] consequences. Its value can be estimated by matching to results of lattice calculations in the Landau gauge [16, 17], and their significance for non-perturbative signatures above the deconfinement phase transition was analyzed in [18]. Chiral quark-model calculations were made in [19] where  $\langle A^2 \rangle$  seems related to constituent quark masses. In spite of all this flagrant need for these unconventional condensates the dynamical origin of  $\langle A^2 \rangle$  remains still somewhat unclear; for recent reviews see, *e.g.*, [20, 21].

For large  $Q^2$  and fixed  $N_c$  the modified OPE (with the  $1/Q^2$  term present) for the chiral combinations of the transverse parts of the vector and axial currents is

$$\begin{aligned}\Pi_{V+A}^T(Q^2) &= \frac{1}{4\pi^2} \left\{ -\frac{N_c}{3} \log \frac{Q^2}{\mu^2} - \frac{\alpha_S}{\pi} \frac{\lambda^2}{Q^2} + \frac{\pi}{3} \frac{\langle \alpha_S G^2 \rangle}{Q^4} + \dots \right\} \\ \Pi_{V-A}^T(Q^2) &= -\frac{32\pi}{9} \frac{\alpha_S \langle \bar{q}q \rangle^2}{Q^6} + \dots\end{aligned}\quad (1)$$

On the other hand, at large- $N_c$  and any  $Q^2$  these correlators may be saturated by infinitely many mesonic states,

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \frac{F_{V,n}^2}{M_{V,n}^2 + Q^2} + c.t., \quad \Pi_A^T(Q^2) = \frac{f^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_{A,n}^2}{M_{A,n}^2 + Q^2} + c.t. \quad (2)$$

The basic idea of parton-hadron duality is to match Eq. (1) and (2) for both large  $Q^2$  and  $N_c$  (assuming that both limits commute). We use the radial Regge spectra, which are well supported experimentally [22]

$$M_{V,n}^2 = M_V^2 + a_V n, \quad M_{A,n}^2 = M_A^2 + a_A n, \quad n = 0, 1, \dots \quad (3)$$

The vector part,  $\Pi_V^T$ , satisfies the once-subtracted dispersion relation

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \left( \frac{F_{V,n}^2}{M_V^2 + a_V n + Q^2} - \frac{F_{V,n}^2}{M_V^2 + a_V n} \right). \quad (4)$$

We need to reproduce the  $\log Q^2$  in OPE, for which only the asymptotic part of the meson spectrum matters. This leads to the condition that at large  $n$  the residues become independent of  $n$ ,  $F_{V,n} \simeq F_V$  and  $F_{A,n} \simeq F_A$ . Thus all the highly-excited radial states are coupled to the current with equal strength! Or: asymptotic dependence of  $F_{V,n}$  or  $F_{A,n}$  on  $n$  would damage OPE. Next, we carry out the sum explicitly (the dilog function is  $\psi(z) = \Gamma'(z)/\Gamma(z)$ )

$$\begin{aligned} \sum_{n=0}^{\infty} \left( \frac{F_i^2}{M_i^2 + a_i n + Q^2} - \frac{F_i^2}{M_i^2 + a_i n} \right) &= \frac{F_i^2}{a_i} \left[ \psi \left( \frac{M_i^2}{a_i} \right) - \psi \left( \frac{M_i^2 + Q^2}{a_i} \right) \right] \\ &= \frac{F_i^2}{a_i} \left[ -\log \left( \frac{Q^2}{a_i} \right) + \psi \left( \frac{M_i^2}{a_i} \right) + \frac{a_i - 2M_i^2}{2Q^2} + \frac{6M_i^4 - 6a_i M_i^2 + a_i^2}{12Q^4} + \dots \right], \end{aligned} \quad (5)$$

where  $i = V, A$ .  $\Pi_{V-A}$  satisfies the unsubtracted dispersion relation (no  $\log Q^2$  term), hence

$$F_V^2/a_V = F_A^2/a_A. \quad (6)$$

This complies to the chiral symmetry restoration in the high-lying spectra [23, 24]. Further, we assume  $a_V = a_A = a$ , or  $F_V = F_A = F$ , which is well-founded experimentally, as  $\sqrt{\sigma_A} = 464 \text{ MeV}$ ,  $\sqrt{\sigma_V} = 470 \text{ MeV}$  [22].

The simplest model we consider has strictly linear trajectories all the way down,

$$\begin{aligned} \Pi_{V-A}^T(Q^2) &= \frac{F^2}{a} \left[ -\psi \left( \frac{M_V^2 + Q^2}{a} \right) + \psi \left( \frac{M_A^2 + Q^2}{a} \right) \right] - \frac{f^2}{Q^2} \\ &= \left( \frac{F^2}{a} (M_A^2 - M_V^2) - f^2 \right) \frac{1}{Q^2} + \left( \frac{F^2}{2a} (M_A^2 - M_V^2)(a - M_A^2 - M_V^2) \right) \frac{1}{Q^4} + \dots \end{aligned}$$

Matching to OPE yields the two Weinberg sum rules:

$$\begin{aligned} f^2 &= \frac{F^2}{a} (M_A^2 - M_V^2), & (\text{WSR I}) \\ 0 &= (M_A^2 - M_V^2)(a - M_A^2 - M_V^2). & (\text{WSR II}) \end{aligned}$$

The  $V + A$  channel needs regularization. We proceed as follows: carry  $d/dQ^2$ , compute the convergent sum, and integrate back over  $Q^2$ . The result is

$$\begin{aligned} \Pi_{V+A}^T(Q^2) &= \frac{F^2}{a} \left[ -\psi \left( \frac{M_V^2 + Q^2}{a} \right) - \psi \left( \frac{M_A^2 + Q^2}{a} \right) \right] + \frac{f^2}{Q^2} + \text{const.} = -\frac{2F^2}{a} \log \frac{Q^2}{\mu^2} \\ &+ \left( f^2 + F^2 - \frac{F^2}{a} (M_A^2 + M_V^2) \right) \frac{1}{Q^2} + \frac{F^2}{6a} (a^2 - 3a(M_A^2 + M_V^2) + 3(M_A^4 + M_V^4)) \frac{1}{Q^4} + \dots \end{aligned}$$

Matching of the coefficient of  $\log Q^2$  to OPE gives the relation

$$a = 2\pi\sigma = \frac{24\pi^2 F^2}{N_c}, \quad (7)$$

where  $\sigma$  denotes the (long-distance) string tension. From the  $\rho \rightarrow 2\pi$  decay one extracts  $F = 154 \text{ MeV}$  [25] which gives  $\sqrt{\sigma} = 546 \text{ MeV}$ , compatible to the value obtained in lattice simulations:  $\sqrt{\sigma} = 420 \text{ MeV}$  [26]. Moreover, from the Weinberg sum rules

$$M_A^2 = M_V^2 + \frac{24\pi^2}{N_c} f^2, \quad a = M_A^2 + M_V^2 = 2M_V^2 + \frac{24\pi^2}{N_c} f^2. \quad (8)$$

Matching higher twists fixes the dimension-2 and 4 gluon condensates:

$$-\frac{\alpha_S \lambda^2}{4\pi^3} = f^2, \quad \frac{\alpha_S \langle G^2 \rangle}{12\pi} = \frac{M_A^4 - 4M_V^2 M_A^2 + M_V^4}{48\pi^2}. \quad (9)$$

Numerically, it gives  $-\frac{\alpha_S \lambda^2}{\pi} = 0.3 \text{ GeV}^2$  as compared to  $0.12 \text{ GeV}^2$  from Ref. [10, 20]. The short-distance string tension is  $\sigma_0 = -2\alpha_S \lambda^2/N_c = 782 \text{ MeV}$ , which is twice as much as  $\sigma$ . The major problem of the strictly linear model is that the dimension-4 gluon condensate is

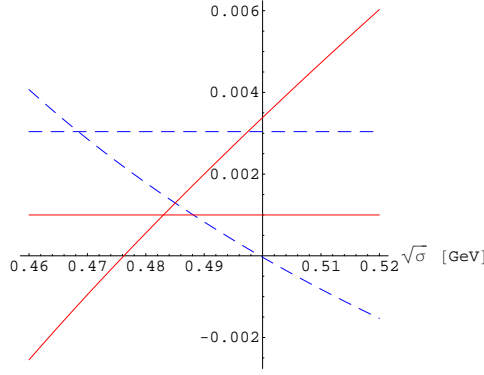


Figure 1: Dimension-2 (solid line, in  $\text{GeV}^2$ ) and -4 (dashed line, in  $\text{GeV}^4$ ) gluon condensates plotted as functions of the square root of the string tension. The straight lines indicate phenomenological estimates. The fiducial region in  $\sqrt{\sigma}$  for which both condensates are positive is in the acceptable range compared to the values of Ref. [22] and other studies.

negative for  $M_V \geq 0.46$  GeV. Actually, it never reaches the QCD sum-rules value. Thus, the strictly linear radial Regge model is *too restrictive*!

We therefore consider a modified Regge model where for low-lying states both their residues and positions may depart from the linear trajectories. The OPE condensates are expressed in terms of the parameters of the spectra. A very simple modification moves only the position of the lowest vector state, the  $\rho$  meson.

$$\begin{aligned} M_{V,0} &= m_\rho, \quad M_{V,n}^2 = M_V^2 + an, \quad n \geq 1 \\ M_{A,n}^2 &= M_A^2 + an, \quad n \geq 0. \end{aligned} \quad (10)$$

For the Weinberg sum rules (we use  $N_c = 3$  from now on)

$$M_A^2 = M_V^2 + 8\pi^2 f^2, \quad a = 8\pi^2 F^2 = \frac{8\pi^2 f^2 (4\pi^2 f^2 + M_V^2)}{4\pi^2 f^2 - m_\rho^2 + M_V^2}. \quad (11)$$

We fix  $m_\rho = 0.77$  GeV, and  $\sigma$  is the only free parameter of the model. Then

$$\begin{aligned} M_V^2 &= \frac{-16\pi^3 f^4 + 4\pi^2 \sigma f^2 - m_\rho^2 \sigma}{4f^2 \pi - \sigma}, \quad -\frac{\alpha_S \lambda^2}{4\pi^3} = \frac{16\pi^3 f^4 - \pi \sigma^2 + m_\rho^2 \sigma}{16f^2 \pi^3 - 4\pi^2 \sigma}, \\ \frac{\alpha_S \langle G^2 \rangle}{12\pi} &= 2\pi^2 f^4 - \pi \sigma f^2 + \frac{3\sigma \left( \frac{m_\rho^2 \sigma}{(\sigma - 4f^2 \pi)^2} - 2\pi \right) m_\rho^2}{8\pi^2} + \frac{\sigma^2}{12}. \end{aligned} \quad (12)$$

The window for which both condensates are positive yields very acceptable values of  $\sigma$ . The consistency check of near equality of the long- and short-distance string tensions,  $\sigma \simeq \sigma_0$ , holds for  $\sqrt{\sigma} \simeq 500$  MeV. The magnitude of the condensates is in the ball park of the “physical” values. The value of  $M_V$  in the “fiducial” range is around 820 MeV. The experimental spectrum in the  $\rho$  channel is has states at 770, 1450, 1700, 1900\*, and 2150\* MeV, while the model gives 770, 1355, 1795, 2147 MeV (for  $\sigma = (0.47 \text{ GeV}^2)$ ). In the  $a_1$  channel the experimental states are at 1260 and 1640 MeV, whereas the model yields 1015 and 1555 MeV.

We note that the  $V - A$  channel well reproduced with radial Regge models. The Das-Mathur-Okubo sum rule gives the Gasser-Leutwyler constant  $L_{10}$ , while the Das-Guralnik-Mathur-Low-Yuong sum rule yields the pion electromagnetic mass splitting. In the strictly linear model with  $M_A^2 = 2M_V^2$  and  $M_V = \sqrt{24\pi^2/N_c} f = 764$  MeV we have  $\sqrt{\sigma} = \sqrt{3/2\pi} M_V = 532$  MeV,  $F = \sqrt{3} f = 150$  MeV,  $L_{10} = -N_c/(96\sqrt{3}\pi) = -5.74 \times 10^{-3} (-5.5 \pm 0.7 \times 10^{-3})_{\text{exp}}$ ,  $m_{\pi^\pm}^2 - m_{\pi^0}^2 = (31.4 \text{ MeV})^2 (35.5 \text{ MeV})_{\text{exp}}^2$ . In our second model with  $\sigma = (0.48 \text{ GeV}^2)$  we find  $L_{10} = -5.2 \times 10^{-3}$  and  $m_{\pi^\pm}^2 - m_{\pi^0}^2 = (34.4 \text{ MeV})^2$ .

To conclude, let us summarize our results and list some further related studies.

- Matching OPE to the radial Regge models produces in a natural way the  $1/Q^2$  correction to the  $V$  and  $A$  correlators. Appropriate conditions are satisfied by the asymptotic spectra, while the parameters of the low-lying states are tuned to reproduce the values of the condensates.

- In principle, these parameters of the spectra are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates.
- Yet, sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, makes such a study difficult or impossible at a more precise level.
- Regge models work very well in the  $V - A$  channel. In [28] it is shown how the spectral (in fact chiral) asymmetry between vector and axial channel is generated via the use of  $\zeta$ -function regularization for *each* channel separately.
- We comment that effective low-energy chiral models produce  $1/Q^2$  corrections (*i.e.* provide a scale of dimension 2), *e.g.*, the instanton-based chiral quark model gives [19]

$$-\frac{\alpha_S}{\pi}\lambda^2 = -2N_c \int du \frac{u}{u + M(u)^2} M(u) M'(u) \simeq 0.2 \text{ GeV}^2. \quad (13)$$

- In the presented Regge approach the pion distribution amplitude is constant,  $\phi(x) = 1$ , at the low-energy hadronic scale, similarly as in chiral quark models [27].

## References

- [1] E. Ruiz Arriola, W. Broniowski, Phys. Rev. D73 (2006) 097502.
- [2] M. Golterman, S. Peris, JHEP 01 (2001) 028.
- [3] S. R. Beane, Phys. Rev. D64 (2001) 116010.
- [4] Y. A. Simonov, Phys. Atom. Nucl. 65 (2002) 135–152.
- [5] M. Golterman, S. Peris, Phys. Rev. D67 (2003) 096001.
- [6] S. S. Afonin, Phys. Lett. B576 (2003) 122–126.
- [7] S. S. Afonin, A. A. Andrianov, V. A. Andrianov, D. Espriu, JHEP 04 (2004) 039.
- [8] S. S. Afonin, Nucl. Phys. B 779 (2007) 13.
- [9] L. S. Celenza, C. M. Shakin, Phys. Rev. D34 (1986) 1591–1600.
- [10] K. G. Chetyrkin, S. Narison, V. I. Zakharov, Nucl. Phys. B550 (1999) 353–374.
- [11] F. V. Gubarev, L. Stodolsky, V. I. Zakharov, Phys. Rev. Lett. 86 (2001) 2220–2222.
- [12] F. V. Gubarev, V. I. Zakharov, Phys. Lett. B501 (2001) 28–36.
- [13] K.-I. Kondo, Phys. Lett. B514 (2001) 335–345.
- [14] H. Verschelde, K. Knecht, K. Van Acoleyen, M. Vanderkelen, Phys. Lett. B516 (2001) 307–313.
- [15] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 74 (2006) 045008.
- [16] P. Boucaud, et al., Phys. Rev. D63 (2001) 114003.
- [17] E. Ruiz Arriola, P. O. Bowman, W. Broniowski, Phys. Rev. D70 (2004) 097505.
- [18] E. Megias, E. Ruiz Arriola, L. L. Salcedo, JHEP 01 (2006) 073.
- [19] A. E. Dorokhov, W. Broniowski, Eur. Phys. J. C32 (2003) 79–96.
- [20] V. I. Zakharov, Nucl. Phys. Proc. Suppl. 164 (2007) 240–247.
- [21] S. Narison, Nucl. Phys. Proc. Suppl. 164 (2007) 225–231.
- [22] A. V. Anisovich, V. V. Anisovich, A. V. Sarantsev, Phys. Rev. D62 (2000) 051502.
- [23] L. Y. Glozman, Phys. Lett. B539 (2002) 257–265.
- [24] L. Y. Glozman, Phys. Lett. B587 (2004) 69–77.
- [25] G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321 (1989) 311.
- [26] O. Kaczmarek, F. Zantow, Phys. Rev. D71 (2005) 114510.
- [27] E. Ruiz Arriola, W. Broniowski, Phys. Rev. D74 (2006) 034008.
- [28] E. Ruiz Arriola, W. Broniowski, Eur. Phys. J. A 31 (2007) 739